

Scale-dependent bias and the halo model

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Abstract

We use a simplified version of the halo model with a power law power spectrum to study scale dependence in galaxy bias at the very large scales relevant to baryon oscillations. In addition to providing a useful pedagogical explanation of the scale dependence of galaxy bias, the model provides an analytic tool for studying how changes in the Halo Occupation Distribution (HOD) impact the scale dependence of galaxy bias on scales $10 < k^{-1} < 1000 h^{-1}\text{Mpc}$, which is useful for interpreting the results of complex N-body simulations. We find that changing the mean number of galaxies per halo of a given mass will change the scale dependence of the bias, but that changing the way the galaxies are distributed within the halo has a smaller effect on the scale dependence of bias at large scales. We use the model to explain the decay in amplitude of the baryon oscillations as k increases, and generalize the model to make predictions about scale dependent galaxy bias when redshift space distortions are introduced.

1 Introduction

The coupling of photons and baryons by Thompson scattering in the early universe results in gravity driven acoustic oscillations of the photon-baryon fluid. The features that appear in both the Cosmic Microwave Background (CMB) anisotropies and matter power spectra are snapshots of the phases of these oscillations at the time of decoupling, and provide important clues used to constrain a host of cosmological parameters. Features in the matter power spectrum, referred to as baryon (acoustic) oscillations, have the potential to strongly constrain the expansion history of the universe and the nature of the dark energy. These features in the matter power spectrum induce correlations in the large-scale clustering of the IGM, clusters or galaxies. Indeed a large,

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high redshift, spectroscopic galaxy survey encompassing a million objects has been proposed (1) as a dark energy probe, building on the successful detection (at low redshift) of the features in the clustering of the luminous red galaxy sample of the Sloan Digital Sky Survey (2). Key to this program is the means to localize the primordial features in the galaxy power spectra, which necessarily involves theoretical understanding of such complex issues as galaxy bias, non-linear structure evolution, and redshift space distortions. These effects can shift the observed scales of peaks and troughs in observations of the baryon oscillations, affecting the transverse and radial measurements differently. Marginalizing over this uncertain scale dependence can be a large piece of the error budget of proposed surveys.

Inroads into modeling galaxy bias, non-linear evolution, and redshift space distortions have been made by several groups using N-body simulations (3), but these simulations are complex and often mask the underlying physical mechanisms. A route to understanding the sophisticated simulations is provided by an analytic tool known as the halo model (4). The halo model makes the assumption that all matter in the universe lives in virialized halos, and that the power spectrum of this matter can be divided up into two distinct contributions; the 1-halo term arising from the correlation of matter with other matter existing in the same halo, and the 2-halo term quantifying the correlation of matter with matter that lives in a different halo. The halo model is extended to calculate galaxy power spectra through the introduction of a Halo Occupation Distribution (HOD), that describes the number of galaxies that exist in a halo of a given mass, and their distribution inside that halo. The HOD will naturally depend on the details of galaxy formation, but is not in itself a theory of galaxy formation. Rather it is a description of the effects of galaxy formation on the number and distribution of the galaxies. The impact of galaxy formation on cosmological observables such as the baryon oscillation can be studied in the halo model by investigating the sensitivity of the observable to changes made to the HOD.

In this paper, we employ a simple version of the analytic halo model to study the origins of scale dependence in galaxy bias. We investigate the impact of changing the HOD on the observed spectra, and show that the scale dependence of the bias arises in a natural way from extending the dark matter description to a description of galaxies. Specifically, in generalizing the description to an ensemble of rare tracers of the dark matter, the 1-halo and 2-halo terms in the power spectrum are shifted by different amounts. The scale dependence in the galaxy bias arises from this difference. We find that for small k , the galaxy power spectrum is sensitive to the number of galaxies that occupy a halo, but not to their positions within the halo. We quantify the impact of redshift space distortions, which find a natural description in the halo model, and discuss the implications for large galaxy redshift surveys.

2 The Halo Model

We will try to understand the scale-dependence of the galaxy bias by examining a simple model. Our goal is not to make precise predictions, but rather to use simple analytic approximations to help interpret the results of N-body simulations, which are much more appropriate for studying the fine details.

Our investigation makes use of the halo model (4), which assumes that all of the mass and galaxies in the universe live in virialized halos, whose clustering and number density is characterized by their mass. This model can be used to approximate the two point correlation function of the mass, and of various biased traces of the mass, such as luminous galaxies. In this framework, there are two contributions to the power spectrum: one arises from the correlation of objects that reside in the same halo (the 1-halo term), and the other comes from the correlation of objects that live in separate halos (the 2-halo term). For the dark matter, for example, the dimensionless power spectrum can be written

$$\Delta_{\text{dm}}^2 \equiv \frac{k^3 P_{\text{dm}}(k)}{2\pi^2} = {}_{1\text{h}}\Delta_{\text{dm}}^2 + {}_{2\text{h}}\Delta_{\text{dm}}^2 \quad (1)$$

where (4):

$${}_{1\text{h}}\Delta_{\text{dm}}^2 = \frac{k^3}{2\pi^2} \frac{1}{\bar{\rho}^2} \int_0^\infty dM n_h(M) M^2 |y(M, k)|^2 \quad (2)$$

$${}_{2\text{h}}\Delta_{\text{dm}}^2 = \Delta_{\text{lin}}^2 \left[\frac{1}{\bar{\rho}} \int_0^\infty dM n_h(M) b_h(M, k) M y(M, k) \right]^2 \quad (3)$$

with M the virial mass of the halo, $\bar{\rho}$ the mean background density, $n_h(M)$ the number density of halos of a given virial mass, and $b_h(M, k)$ the halo bias (5). The function $y(M, k)$ is the Fourier transform of the halo profile which describes how the dark matter is spatially distributed within the halo.

Expressed this way, Eqs. (2) and (3) lend themselves to a fairly intuitive interpretation. In the two halo term, the dark matter being correlated lives in widely separated halos, of different masses. For each of the two halos, the mass is multiplied by the function y that governs its spatial distribution within a halo, weighted by the number density of halos n_h with bias b_h , and integrated over all possible halo masses. The one halo term is even simpler – correlating two bits of dark matter residing in the same halo. Thus there are two factors of $M \times y$ weighted with the number density of halos n_h , and integrated over all halo masses.

We can generalize this framework to compute the 1- and 2-halo terms for a galaxy population that traces the density field (4). We divide the galaxies into

two sub-populations, centrals and satellites. The centrals will reside at the center of the host halo, while the satellites will trace the dark matter. The Halo Occupation Distribution (HOD) sets the number of tracers in a halo of mass M . We assume there is either a central galaxy or not, and the number of satellites is Poisson distributed (6). For our model we will use

$$\langle N_c \rangle = \Theta(M - M_{\min}) \quad (4)$$

$$\langle N_s \rangle = \Theta(M - M_{\min}) \left(\frac{M}{M_{\text{sat}}} \right)^a \quad (5)$$

where Θ is the Heaviside function, and $M_{\min} < M_{\text{sat}}$. Note that the central galaxies do not trace the halo profile, and are not weighted by y . The generalization of the 1-halo and 2-halo terms is given by (4)

$${}_{2h}\Delta_g^2 = \Delta_{\text{lin}}^2 \left[\frac{1}{\bar{n}_g} \int_{M_{\min}}^{\infty} dM n_h(M) b_h(M) \left(1 + \left(\frac{M}{M_{\text{sat}}} \right)^a y(M, k) \right) \right]^2 \quad (6)$$

$${}_{1h}\Delta_g^2 = \frac{k^3}{2\pi^2} \frac{1}{\bar{n}_g^2} \int_{M_{\min}}^{\infty} dM n_h(M) \left(2 \left(\frac{M}{M_{\text{sat}}} \right)^a y(M, k) + \left(\frac{M}{M_{\text{sat}}} \right)^{2a} |y(M, k)|^2 \right) \quad (7)$$

where

$$\bar{n}_{\text{gal}} = \int_{M_{\min}}^{\infty} dM n_h(M) \left(1 + \left(\frac{M}{M_{\text{sat}}} \right)^a \right) \quad (8)$$

is the number density of galaxies. The interpretation of these expressions is similar to that of the dark matter.

For the purposes of this toy model, we shall adopt a power law for the linear power spectrum,

$$\Delta_{\text{lin}}^2 = \left(\frac{k}{k_*} \right)^{3+n} = \kappa^{3+n} \quad (9)$$

and define a dimensionless wavenumber $\kappa \equiv k/k_*$. In order to simplify many of the expressions in the calculation, we find it useful to change variables from M to a dimensionless quantity, ν , related to the peak height of the overdensity. For the power law model considered here ν is a simple function of the mass:

$$\nu(M) = \left(\frac{\delta_c}{\sigma(M)} \right)^2 = \left(\frac{M}{M_*} \right)^{(n+3)/3} = m^{(n+3)/3} \quad . \quad (10)$$

Here, $\delta_c = 1.686$ and $\sigma(M)$ is the linear theory variance in top hat spheres of radius $R = (3M/4\pi\bar{\rho})^{1/3}$. We have introduced a dimensionless mass m in terms of the scale mass M_* , the mass for which $\sigma(M_*) = \delta_c$ and $\nu = 1$. Note M_* is a function of the power spectrum normalization k_* and the index n , and

can be computed using the relation

$$\sigma^2(R) = \int_0^\infty \frac{dk}{k} \Delta_{\text{lin}}^2 \left[\frac{3j_1(kR)}{kR} \right]^2 \quad (11)$$

where j_1 is the spherical Bessel function of order 1.

The mass function $n_h(M)$ takes a simple form when expressed in terms of the multiplicity function, $f(\nu)$. The multiplicity function is a normalized number density of halos at a given mass:

$$f(\nu)d\nu = \frac{M}{\bar{\rho}} n_h(M) dM \quad \text{with} \quad \int f(\nu) d\nu = 1 \quad . \quad (12)$$

For the Press-Schechter (P-S) mass function (7)

$$f(\nu) = \frac{e^{-\nu/2}}{\sqrt{2\pi\nu}} \quad (13)$$

and the halos form biased tracers of the density field. On large scales, for small fluctuations, the bias is (5)

$$b_h(\nu) = 1 + \frac{\nu - 1}{\delta_c} \quad (14)$$

which satisfies $\int d\nu f(\nu)b(\nu) = 1$. In detail of course the halos do not provide a linearly biased tracer of the linear density field. On smaller scales both higher order terms and halo exclusion effects give rise to a scale-dependence. Both analytic calculations (5) and simulations (3) suggest that this is a few percent correction on the scales of interest to us and we shall henceforth neglect it.

When looking at large scales, such as those relevant to the baryon wiggles in the linear power spectrum, the function $y(M, k)$ can be accurately approximated by a Taylor expansion into powers of kr_v , where r_v is the virial radius, which depends upon the mass. Assuming an NFW form (10) for $y(M, k)$ and expressing the mass dependence explicitly, the expression is

$$\begin{aligned} y(M, k) &= 1 + c_2 (kr_v)^2 + c_4 (kr_v)^4 + \dots \\ &= 1 + c_2(k_\star r_\star)^2 \kappa^2 m^{2/3} + c_4(k_\star r_\star)^4 \kappa^4 m^{4/3} + \dots \end{aligned} \quad (15)$$

Here we have introduced another quantity, the virial radius of an M_\star halo, $r_\star \equiv r_v(M_\star) = (3M_\star/4\pi\Delta\bar{\rho})^{1/3}$, where Δ is the virialization overdensity, which we will take to be $\Delta = 200$. The expansion coefficients c_2 and c_4 are functions of the halo concentration, and for the NFW model are ratios of gamma functions. For cluster sized halos we expect $c \simeq 5$ which leads to $c_2 \simeq -0.049$ and $c_4 \simeq 0.0014$, while for galaxies $c \simeq 10$ making $c_2 \simeq -0.04$ and $c_4 \simeq 0.0011$. The quantity $k_\star r_\star$ can be computed using the relation in Eq. (11), and turns

n	$k_\star r_\star$		A_\star		γ_{dm}
0	$\Delta^{-1/3} (3\pi/2\delta_c^2)^{1/3}$	0.2023	δ_c^{-2}	0.3518	1
$-\frac{1}{2}$	$\Delta^{-1/3} (12\sqrt{\pi}/7\delta_c^2)^{2/5}$	0.1756	$8/7 (12/7\pi^2)^{2/5} \delta_c^{-12/5}$	0.2299	1.18
-1	$\Delta^{-1/3} (3/2\delta_c)$	0.1521	$(9/4\pi) \delta_c^{-3}$	0.1494	1.60
$-\frac{3}{2}$	$\Delta^{-1/3} (16\sqrt{\pi}/15\delta_c^2)^{2/3}$	0.1303	$(512/675) \delta_c^{-4}$	0.0939	3
-2	$\Delta^{-1/3} (3\pi/5\delta_c^2)$	0.1134	$(18\pi^2/125) \delta_c^{-6}$	0.0619	15

Table 1

Expressions and values for $k_\star r_\star$ and A_\star in terms of $\delta_c = 1.686$ and the virialization overdensity $\Delta = 200$. Here k_\star is the normalization of the dark matter power spectrum, and $r_\star = r_v(M_\star)$ is the virial radius of a halo of mass M_\star . The factor $A_\star = k_\star^3 M_\star / (2\pi^2 \bar{\rho})$ relates the amplitude of the 1- and 2-halo terms and γ_{dm} is defined in Eq. (18).

out to be a function only of the index n . We have tabulated the expressions and values of $k_\star r_\star$ in Table 1. On large scales, where $k < k_\star$, we see that the coefficients of the last two terms in the expression for $y(M, k)$ are extremely small. We have repeated our analysis neglecting these terms and find that these are insignificant corrections to the scale dependence we are studying. This is consistent with the results of (8), who found that the local relation between galaxies and mass within a halo does not significantly impact the large scale galaxy correlation function. For simplicity we shall set $y(M, k) = 1$ for the remainder of the paper.

3 Results – real space

Having argued that we can safely approximate y and b_h as scale independent quantities when studying clustering at large scales our expressions simplify dramatically. The mass power spectrum is simply³

$$\Delta_{\text{dm}}^2(k) = \kappa^3 (\kappa^n + A_\star \gamma_{\text{dm}}) \quad \kappa \ll 1 \quad (16)$$

where A_\star and γ_{dm} are n -dependent constants

$$A_\star = \frac{k_\star^3 M_\star}{2\pi^2 \bar{\rho}} \quad (17)$$

$$\gamma_{\text{dm}} = \int_0^\infty m(\nu) f(\nu) d\nu = 2^{3/(3+n)} \pi^{-1/2} \Gamma [1/2 + 3/(n+3)] \quad (18)$$

We list the values of A_\star and γ_{dm} for some values of n in Table 1. Referring to the Table we see that, for n near -1, the 1-halo term dominates only for

³ If appropriate halo profiles, e.g. Gaussians, are chosen the full k -dependent integrals can also be done in terms of special functions.

$k > k_*$, outside of the range of relevance for us.

The scale-dependent bias can be defined as

$$B^2(k) \equiv \frac{^{2\text{h}}\Delta_g^2 + {}_{1\text{h}}\Delta_g^2}{^{2\text{h}}\Delta_{\text{dm}}^2 + {}_{1\text{h}}\Delta_{\text{dm}}^2} \quad (19)$$

which can be re-written to explicitly exhibit its scale dependence as

$$B^2(k) = \left(\frac{1}{\alpha_g^2} \right) \frac{\beta_g^2 + A_* \gamma_g \kappa^{-n}}{1 + A_* \gamma_{\text{dm}} \kappa^{-n}} \quad (20)$$

$$\simeq (\beta_g/\alpha_g)^2 (1 + \zeta \kappa^{-n} + \dots) \quad (21)$$

where α_g , β_g and γ_g are dimensionless integrals of ν , α_g is the galaxy number density in dimensionless units, β_g/α_g is the galaxy weighted halo bias and γ_g counts⁴ the number of galaxy pairs in a single halo⁴. We have neglected terms higher order in κ . The term κ^{-n} encodes the leading order scale dependence and is proportional to the inverse of the linear dark matter power spectrum. Choosing $a = 1$ in our HOD as a representative example the relevant integrals are

$$\alpha_g = \int_{\nu_{\min}}^{\infty} m(\nu)^{-1} f(\nu) [1 + m(\nu)/m_{\text{sat}}] d\nu \quad (22)$$

$$\beta_g = \int_{\nu_{\min}}^{\infty} m(\nu)^{-1} f(\nu) [1 + m(\nu)/m_{\text{sat}}] b_h(\nu) d\nu \quad (23)$$

$$\gamma_g = \int_{\nu_{\min}}^{\infty} m(\nu)^{-1} f(\nu) [2m(\nu)/m_{\text{sat}} + (m(\nu)/m_{\text{sat}})^2] d\nu \quad (24)$$

and the factor governing the scale-dependence is

$$\zeta(\nu_{\min}, m_{\text{sat}}, n) = A_* (\gamma_g/\beta_g^2 - \gamma_{\text{dm}}) \quad (25)$$

Note ζ depends on the number of pairs of galaxies divided by the square of the large-scale bias. If one wishes to reintroduce the halo profiles there is a simple modification to the integrals. In α_g , β_g , and γ_g , every occurrence of $m(\nu)/m_{\text{sat}}$ will be multiplied by $y(\nu, k)$. For the dark matter, γ_{dm} will have an extra factor of $|y(\nu, k)|^2$ in the integrand, and the 1 in the denominator of Eq. (20) will be replaced by $\int b_h(\nu) f(\nu) y(\nu, k) d\nu$ squared.

In this form it is clear that the scale-dependent bias arises because the 1- and 2-halo terms for the galaxies are different multiples of their respective dark matter counterparts. Typically the 1-halo term is enhanced more than the 2-halo term, leading to an increase in the bias with decreasing scale. A cartoon of this is shown in Fig. 1. The relative enhancements of the 1 and 2-halo terms depend on the HOD parameters for the galaxy population used

⁴ In the limit $y(\nu, k) = 1$ we have $\alpha_{\text{dm}} = \beta_{\text{dm}} = 1$. The expression for γ_{dm} is given in Eq. (18).

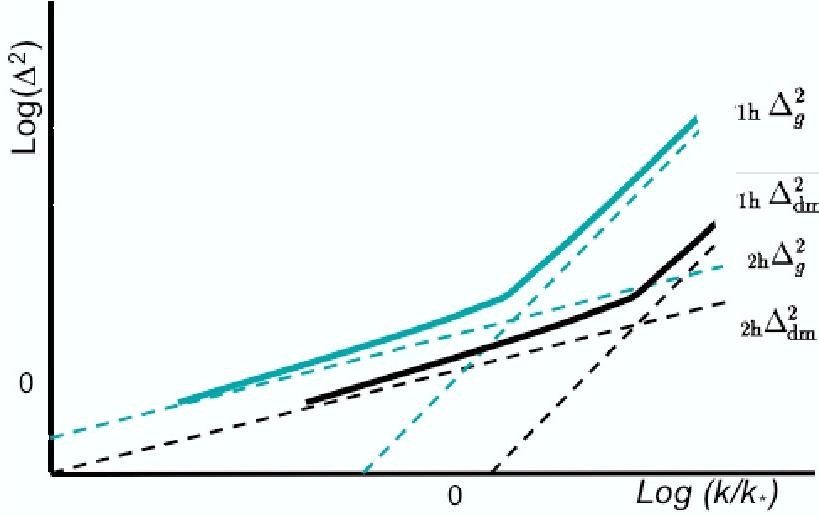


Fig. 1. A cartoon illustrating the difference in the shift of the 1-halo and 2-halo terms (dashed lines) of the galaxy power spectra with respect to the dark matter. Because of this difference, the 1-halo term dominates on larger scales for the galaxy spectrum. This leads to a change in the ratio of the total power (solid curves), which leads to a scale dependent galaxy bias.

as tracers. Note also that in our simple model the 2-halo term retains the oscillations of the linear theory spectrum, while in the 1-halo term they are absent. This provides a partial⁵ explanation of the reduction of the contrast of the oscillations with increasing k .

Figure 2 shows ζ vs. ν_{\min} for several different values of the other HOD parameter M_{sat} . We see that the scale dependence is more prominent as the number of satellite galaxies is increased, and that a higher threshold halo mass for containing a central galaxy leads to a more scale dependent bias at large scales. At fixed number density ζ increases with increasing bias and is more rapidly increasing for rarer objects. At fixed bias ζ is larger the rarer the object. Our model is not sophisticated enough to expect good agreement with large N-body simulations, however comparing to the work of (3) we find good qualitative agreement in the scale-dependence of the bias for $0.01 < k/(h \text{ Mpc}^{-1}) < 0.1$.

We note that the 1- and 2-halo decomposition leads us to a new parameterization of the scale-dependent bias. In the limit where halo profiles and scale-dependent halo bias can be neglected the most natural description of

⁵ In a more complex/realistic model the 2-halo term involves the non-linear power spectrum and non-linear bias of the tracers, including halo exclusion effects. Mode coupling thus appears to reduce the baryon signal even at the 2-halo level.

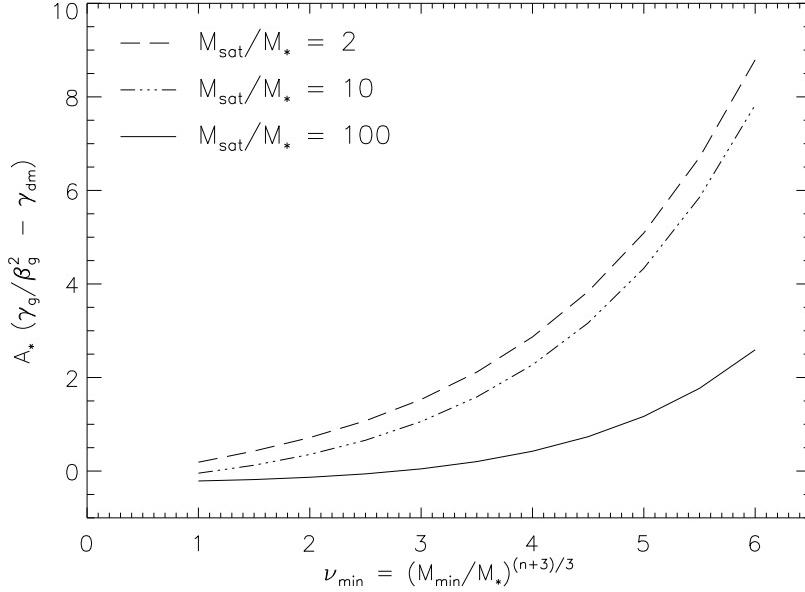


Fig. 2. The factor ζ that governs the strength of the scale dependent part of the galaxy bias.

the galaxy spectrum is

$$\Delta_g^2 = b^2 \Delta_{\text{lin}}^2(k) + \left(\frac{k}{k_1}\right)^3 \quad (26)$$

which has two free parameters, b and k_1 . We expect this will describe the largest part of the scale-dependent bias. Non-linear bias, halo exclusion and profiles will show up as smaller corrections to this formula, such as a scale-dependence in b . It is difficult to compare the scale dependence in this framework to other treatments of scale dependent bias (e.g. (11)) where the galaxy density contrast is expanded in moments of the matter density contrast (b_1 , b_2 , etc.), because the matter density contrast itself has both 1-halo and 2-halo contributions, and furthermore we have not extended our analysis to the bispectrum or higher order.

4 Results – redshift space

Keeping to our philosophy of examining qualitative behavior in a simple model, we can extend these results to redshift space. The 2-point function in redshift space differs from that in real space due to two effects (12). The first, effective primarily on very large scales, accounts for the fact that dark matter and the galaxies that trace it have a tendency to flow toward over-densities as the structure in the universe is being assembled, enhancing the fluctuations in redshift space (9). The second comes into play inside virial-

ized structures, where random motions within the halo reduce fluctuations in redshift space.

These corrections impact the 1-halo and 2-halo terms. The inflow effect primarily impacts the 2-halo term while virial motions primarily affect small scales which are dominated by the 1-halo term (13). The boost in the observed density contrast, δ_k , due to instreaming is given⁶ by $(1 + f\mu^2)$ where $\mu = \hat{r} \cdot \hat{k}$ and $f \simeq \Omega_m^{0.6}$ (9). The small scale suppression we take to be Gaussian. In general, when extending the model to galaxies tracing the dark matter, one should distinguish between central and satellite galaxies, since the latter have much larger virial motions and will therefore suffer more distortion in redshift space. We approximate this by taking $\sigma_{v,\text{cen}}^2 \approx 0$ and $\sigma_{v,\text{sat}}^2 = GM/2r_{\text{vir}}$. Converting from velocity to distance we have for an M_\star halo $\sigma_\star \rightarrow \sqrt{\Delta/4} r_\star \simeq 7 r_\star$. Defining $y_s(\nu, k) = y(\nu, k) e^{-(k\sigma_v\mu)^2/2}$ the 1- and 2-halo terms are then given by (13)

$${}_{1h}\Delta_{\text{dm}}^2 = \frac{k^3}{2\pi^2} \frac{M_\star}{\bar{\rho}} \int_0^\infty m(\nu) f(\nu) |y_s(\nu, k)|^2 d\nu \quad (27)$$

$${}_{2h}\Delta_{\text{dm}}^2 = \Delta_{\text{lin}}^2 \left[\int_0^\infty f(\nu) (1 + f\mu^2) b_h(\nu) y_s(\nu, k) d\nu \right]^2 \quad (28)$$

for the dark matter and

$$\begin{aligned} {}_{1h}\Delta_g^2 &= \frac{k^3}{2\pi^2} \frac{\bar{\rho}}{n_g^2 M_\star} \int_{\nu_{\min}}^\infty m^{-1}(\nu) f(\nu) \\ &\quad \left[2 \left(\frac{m(\nu)}{m_{\text{sat}}} \right) y_s(\nu, k) + \left(\frac{m(\nu)}{m_{\text{sat}}} \right)^2 |y_s(\nu, k)|^2 \right] d\nu \end{aligned} \quad (29)$$

$$\begin{aligned} {}_{2h}\Delta_g^2 &= \Delta_{\text{lin}}^2 \left[\frac{\bar{\rho}}{n_g M_\star} \int_{\nu_{\min}}^\infty m^{-1}(\nu) f(\nu) b_h(\nu) \left(1 + \frac{m(\nu)}{m_{\text{sat}}} y_s(\nu, k) \right) d\nu + \right. \\ &\quad \left. f\mu^2 \int_0^\infty f(\nu) b_h(\nu) y_s(\nu, k) d\nu \right]^2 \end{aligned} \quad (30)$$

for the galaxies. As discussed in (13), in the 2-halo term the effect of peculiar velocities, going as $f\mu^2$, is governed by the mass rather than the galaxy density field, requiring the addition of a separate integral over ν . Conceptually, this term is added to account for extra clustering in redshift space induced by the bulk peculiar flow of the galaxies in one halo under the influence of the dark matter in other halos. For some purposes it is useful to average over

⁶ It has been argued in Ref. (14) that the form $1 + f\mu^2$ is not highly accurate on the scales relevant to observations and higher order corrections apply. Since it is our intent to gain qualitative understanding rather than quantitative accuracy we shall use the simplest form: $1 + f\mu^2$. Deviations from this will be yet another source of scale-dependence, but numerical simulations suggest it is small.

orientations of the galaxy separations (i.e. integrate over μ) but in the case of studying baryon oscillations, doing so throws away valuable information.

As before we note that $y(\nu, k) \approx 1$ and $\exp[-(k\sigma_v\mu)^2/2] \approx 1$ for $k \ll k_*$ so the effect of redshift space distortions is primarily to enhance the 2-halo term – this makes the power spectrum “more linear” in redshift space than real space. However the second of our approximations, $\exp[-(k\sigma_v\mu)^2/2] \approx 1$, is not as good as the first, $y(\nu, k) \approx 1$, so there is enhanced k -dependence from the individual terms. For the interesting range of n , $k_*\sigma_* \sim 1$ so the exponential can only be neglected when $\kappa^2\nu^{2/(n+3)} \ll 1$ for all values of ν that significantly contribute to the integral; i.e. near the peak of the integrand. For example, we see scale dependence at smaller k in the 1-halo term in redshift space than in real space. At $\kappa = 1/2$, the exponential term induces a 13-14% change in the 1-halo terms along the line of sight for both the dark matter and a moderately biased sample of galaxies, leading to a percent level correction in the ratio of power spectra. The error decreases rapidly as $|\mu|$ decreases. The importance of the exponential factor depends somewhat on the HOD parameters. The correction to the galaxy 1-halo term is larger as M_{\min} increases, but is smaller as M_{sat} increases, due to the decreasing number of satellite-satellite pairs.

For completeness we write the scale-dependent bias in redshift space in the approximation that $y_s(\nu, k) \simeq 1$.

$$B^2(k, \mu) = \left(\frac{1}{\alpha_g^2} \right) \frac{[\beta_g + \alpha_g f \mu^2]^2 + \kappa^{-n} A_* \gamma_g}{[1 + f \mu^2]^2 + \kappa^{-n} A_* \gamma_{\text{dm}}} \quad (31)$$

$$\simeq \frac{\beta_g^2}{\alpha_g^2} \frac{\Xi_g^2}{\Xi_{\text{dm}}^2} \left(1 + A_* \kappa^{-n} \left[\frac{\gamma_g}{\Xi_g^2} - \frac{\gamma_{\text{dm}}}{\Xi_{\text{dm}}^2} \right] + \dots \right) \quad (32)$$

where we have defined $\Xi_g = 1 + (\alpha_g f / \beta_g) \mu^2$ and $\Xi_{\text{dm}} = 1 + f \mu^2$ to simplify the equations.

5 Conclusions

Models of structure formation where $\Omega_b \not\propto \Omega_m$ predict a series of features in the linear theory matter power spectrum, akin to the acoustic peaks seen in the angular power spectrum of the cosmic microwave background. These peaks provide a calibrated standard ruler, and a new route to constraining the expansion history of the universe. In order to realize the potential of this new method, we need to understand the conversion from what we measure – the non-linear galaxy power spectrum in redshift space – to what the theory unambiguously provides – the linear theory matter power spectrum in real space. The ability of N-body simulations to calibrate this mapping is improving

rapidly, but the complexity of the simulations can often mask the essential physics. In this paper we have tried to investigate the issues using a simplified model which can give qualitative insights into the processes involved.

In our toy model we find that the distribution of galaxies within halos and the complexities of scale-dependent halo bias are sub-dominant contributions to the scale-dependence of galaxy bias. The dominant effect is the relative shifts of the 1- and 2-halo terms of the galaxies compared to the matter. The amplitude of the scale dependent bias on very large scales is parameterized by a quantity, ζ , which depends on the galaxy HOD. For our two parameter HOD we find ζ increases with increasing bias at fixed number density and is more rapidly increasing for rarer objects. At fixed bias, ζ is larger the rarer the object.

The 1- and 2-halo decomposition leads us to a new parameterization of the scale-dependent bias. In the limit where halo profiles and scale-dependent halo bias can be neglected the most natural description of the galaxy spectrum is

$$\Delta_g^2 = b^2 \Delta_{\text{lin}}^2(k) + \left(\frac{k}{k_1}\right)^3 \quad (33)$$

which has two free parameters, b and k_1 . This is very close to the phenomenologically motivated form proposed by (15). The extra k -dependence these authors allowed in their multiplicative and additive terms can be understood here as the effect of non-linear power, non-linear bias and halo exclusion and halo profiles. The corrections appear first in the 2-halo term and then at smaller scales in the 1-halo term. Our results also suggest that on very large scales, the bias in configuration space has relatively little scale dependence because the effects of the 1-halo term are strictly limited to scales smaller than the virial radius of the largest halo.

We would like to thank D. Eisenstein and R. Sheth for conversations. The simulations referred to in this work were performed on the IBM-SP at NERSC. This work was supported in part by NASA and the NSF.

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